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The self-dual string and anomalies in the M5-brane

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Abstract: We study the anomalies of a charge Q_2 self-dual string solution in the Coulomb branch of Q_5 M5-branes. Cancellation of these anomalies allows us to determine the anomaly of the zero-modes on the self-dual string and their scaling with Q_2 and Q_5. The dimensional reduction of the five-brane anomalous couplings then lead to certain anomalous couplings for D-branes.

Keywords: M-Theory, Anomalies in Field and String Theories.

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Contents

1. Introduction

There remain many puzzling aspects concerning coincident branes in M-theory. One of the central puzzles involves the lack of a microscopic derivation of the number of degrees of freedom on Q_5 coincident five-branes. Some information about the theory has been obtained through circuitous methods such as anomalies [1], low energy scattering [2] and the AdS/CFT correspondence [3]. All these methods show that in the large Q_5 limit the number of degrees of freedom scale as Q_5^3 for the five-brane. For the two-brane the number of degrees of freedom scales like $Q_2^{3/2}$.

In the five-brane itself there are self-dual strings [4]–[6]. These appear as solitonic solutions to the nonlinear five-brane theory and are associated with the five-brane world-volume description of a membrane ending on a five-brane. From the point of view of a spontaneously broken five-brane theory, these strings are similar to W-bosons, becoming tensionless when the five-brane separation of the branes vanishes. There is no adequate description of these self-dual strings. Apart from the problem of describing tensionless string dynamics, self-dual strings are never weakly coupled and so one cannot use standard perturbative techniques. Furthermore, the five-brane theory in which these strings live has no known description when there is more than one five-brane.

This paper will be concerned with the study of these strings, in particular, with Q_2 coincident self-dual strings. In [7] it was found that the absorption cross section of Q_2 coincident self-dual strings (when the number of five-branes $Q_5 = 1$) is proportional to Q_2 indicating that number of degrees of freedom scales linearly with Q_2. A similar scattering calculation was done for a membrane probing a supergravity solution describing Q_5 coin-

JHEP11(2004)015

cident five-branes, indicating that the number of degrees of freedom scale as Q_5 (this is with $Q_2 = 1$). (Note, the same reasoning applied to D3 branes where the absorption cross section for Q D3 branes was shown to scale as Q^2 indicates a Q^2 scaling in the number of light degrees of freedom as is expected for Q coincident three-branes [2].)

Here we wish to consider a more general situation and use anomalies to determine the scaling of the number of degrees of freedom for Q_2 self-dual strings in a generic spontaneously broken five-brane theory associated with Q_5 five-branes in the Coulomb branch. We will be interested in the dependence of the anomaly on the charges Q_2 and Q_5 as well as the role of the unbroken gauge group. The power of anomalies is that they are topological in nature and can be studied in the low-energy theory, yet provide a probe into high energy physics. This will prove very powerful in this situation where a description of the fundamental degrees of freedom is lacking and we have only an effective low energy description.

We will consider a five-brane theory labelled by an ADE Lie algebra (how this symmetry is realised in terms of local fields is not known (see [8] for a discussion of this problem). The simplest case is the $\mathrm{U}(N)$ theory, which is actually a $\mathrm{U}(1)\times\mathrm{SU}(N)$ theory with the decoupled $\mathrm{U}(1)$ corresponding to the Nambu-Goldstone mode of translating the whole stack of branes. Spontaneous symmetry breaking of the theory occurs when one (or more) of the branes are separated off from the stack. There will then be a $\mathrm{U}(1)$ mode corresponding to fluctuations in the separation of the brane stacks. At low energies, i.e. at scales less than the inverse brane separation, that $\mathrm{U}(1)$ mode will be described by a $\mathrm{U}(1)(0,2)$ tensor multiplet.

Self-dual strings are solutions of the (0,2) abelian tensor multiplet. As such we can embed the known self-dual string solution into the $\mathrm{U}(1)$ tensor multiplet corresponding to this separation mode as opposed to the usual overall $\mathrm{U}(1)$ translation mode. This will allow us to investigate properties of the spontaneously broken five-brane theory. In particular, we will be able to see how anomalies in the normal bundle of the self-dual string may be cancelled by inflow from Wess-Zumino type terms in the five-brane world volume theory. Imposing this cancellation will then allow us to determine the scaling of the coefficient in terms of Q_2 and Q_5 and the dependence on the unbroken gauge group. The inflow mechanism is analogous to the sort used to cancel anomalies of intersecting D-branes [9]–[11] or the M-theory five-brane itself, [1, 12] with an inflow from the supergravity bulk.

Now the *bulk* is the five-brane world volume and the defect is the self-dual string. Similar issues concerning the anomalies of self-dual strings have been discussed in [13, 14]; here we expand their discussion to multiple branes and relate the couplings involved in anomaly cancellation to interactions discussed in the recent literature [15, 16]. While this paper was in preparation we became aware of [17] which also considers anomaly cancellation for strings in a six dimensional theory but does not consider the anomalies normal to the five-brane nor allow for a scaling of the zero modes with the charge.

Apart from providing an insight into the self-dual string, the terms that are required by anomaly inflow on the five-brane imply certain anomalous couplings for D-branes via dimensional reduction. These terms are closely related to terms discussed elsewhere in the literature [18, 19] and will be discussed in the penultimate section of the paper.

JHEP11(2004)015

2. The self-dual string fermion zero modes and their anomalies

2.1 The self-dual string solution

The Bosonic field content of the 5+1 dimensional (0,2) tensor multiplet theory consists of a two form field b whose three form field strength h obeys a self-duality constraint [20] and five scalar fields, ϕ^i, $i = 1, \ldots, 5$. The self-dual string [6] is a half BPS solution with the two form b field excited along with a single scalar field, denoted here as ϕ. The solution is given explicitly by:

$$\phi(r) = \phi_0 + \frac{2Q}{r^2} \quad h_{01p}(r) = \pm\frac{1}{4}\partial_p\phi(r) \quad h_{mnp}(r) = \pm\frac{1}{4}\epsilon_{mnpq}\partial_q\phi(r) \quad m,n,p,q = 2,3,4,5\,. \tag{2.1}$$

r is the radial coordinate of the space transverse to the string i.e. for a string lying along x^1, $r^2 = x_2^2 + x_3^2 + x_4^2 + x_5^2$ and ϵ_{mnpq} is the associated epsilon tensor of this transverse space. The charge of the string is, $Q_2 = \mp Q$.

This solution can be viewed as the worldvolume description of an M2 or anti-M2 brane ending on the M5-brane. The field ϕ represents the value of one of the coordinates transverse to the M5. With the convention that ϕ increases from left to right, the solution with the upper choice of $\pm$ sign and $Q > 0$ corresponds to an M2 coming in from the right and terminating on the $M5$ located at ϕ_0. The lower choice of sign with $Q > 0$ is then an anti-M2 coming in from the right while the upper choice of sign with $Q < 0$ is an M2 coming in from the left and the lower choice of sign with $Q < 0$ is an anti-M2 coming in from the left.

2.2 Fermion zero modes

This self-dual string solution preserves eight supercharges (one half BPS with respect to the five-brane and one quarter BPS with respect to M-theory). As usual for such BPS objects, the fermion zero modes of the lowest charge solution are generated by the broken supersymmetries, that is by $\epsilon^{\alpha l}$ which satisfy:

$$\epsilon^{\beta j} = \mp(\gamma^{01})^{\beta}_{\alpha}(\gamma_{5'})^{j}_{i}\epsilon^{\alpha i}\,, \tag{2.2}$$

where $\alpha, \beta = 1 \ldots 4$ are (Weyl) spinor indices of Spin$(1,5)$ and $i, j = 1 \ldots 4$ are spinor indices of USp(4) the Spin cover of the SO(5) R-symmetry group. The choice of plus or minus is associated with the choice of sign in the solution in (2.1) . We now wish to decompose these Fermionic zero modes into representations of:

$$\mathrm{Spin}(1,1) \times \mathrm{Spin}(4)_T \times \mathrm{Spin}(4)_N\,. \tag{2.3}$$

This decomposition is the Spin cover of the Lorentz group that is preserved by the self-dual string solution. (The original SO$(1,5) \times$ SO(5) group which is preserved by the five-brane becomes broken by the self-dual string solution to SO$(1,1) \times$ SO$(4)_T \times$ SO$(4)_N$). The subscripts T, N denote tangent and normal to the five-brane world volume respectively. An eigenspinor of γ^{01} will be a 1+1 chiral spinor and an eigenspinor of $\gamma_{5'}$ will be a Weyl spinor of the $Spin(4)_N$. Importantly, the 6d Weyl spinors that are also eigenspinors of

γ^{01} are Weyl spinors of $\mathrm{Spin}(4)_T$. Putting these facts together implies the BPS self-dual string has (4,4) supersymmetry in 1+1 dimensions with the Fermions lying in the following representations of (2.3) :

$$(2,1,1,2)^{1/2} \oplus (1,2,2,1)^{-1/2} \,, \tag{2.4}$$

and the anti-BPS (negative sign in (2.1)) string has the Fermions lying in:

$$(2,1,2,1)^{1/2} \oplus (1,2,1,2)^{-1/2} \,. \tag{2.5}$$

The superscript labels the $\mathrm{SO}(1,1)$ helicity with the numbers in brackets labelling the $\mathrm{Spin}(4) = \mathrm{SU}(2) \times \mathrm{SU}(2)$ representations.

For solutions with charge $|Q| \geq 1$ there will be $|Q|$ such multiplets. One can argue for this result in several ways. Since the self-dual string solution is BPS, solutions with $|Q| > 1$ can be deformed into $|Q|$ separated string solutions, each with its own centre of mass Bosonic zero modes. Supersymmetry then requires that each of these Bosonic multiplets be accompanied by fermion zero modes. Alternatively, we can reduce this system to a D-brane configuration (as will be discussed in detail later) and then the $|Q|$ fermion zero modes arise from the usual Chan-Paton factors. In principle it should also be possible to show this by analysing an index theorem for the corresponding Dirac operator on the brane, but to our knowledge this has not been done in the literature. (It would be interesting to see this explicitly since the Dirac operator on a brane in the presence of a background field is somewhat different from the usual Dirac operator).

The above analysis has all been concerned with $Q_5 = 1$, that is membranes ending on a single M5-brane. For $Q_5 > 1$ the zero mode structure on strings corresponding to membranes ending on the stack of fivebranes is unknown. The main result of this paper will be to put constraints on the zero mode structure for this case.

2.3 Zero mode anomalies

We now turn to a computation of the anomaly of the fermion zero modes for $Q_5 = 1$. We are interested in anomalies in diffeomorphisms of the eleven-dimensional spacetime which preserved the self-dual string solution of the M5-brane, or equivalently which preserve the configuration of M2-branes ending on the M5-brane. These diffeomorphisms act as diffeomorphisms of the string world-sheet, or as gauge transformations of the $\mathrm{SO}(4)_T \times \mathrm{SO}(4)_N$ normal bundle to the string. Since there are equal numbers of left and right-movers, there is no anomaly in world-sheet diffeomorphisms. However, the left and right moving fermions are in different representations of the normal bundle (the R-symmetry group) which will give rise to a normal bundle anomaly. (In a field theory context these would be the 't Hooft anomalies).

The anomaly can be computed by treating the each $\mathrm{SO}(4)$ symmetry as a gauge symmetry (see e.g. the discussion in [12]). In two dimensions the anomaly is derived by descent from a four-form characteristic class. $\mathrm{SO}(4)$ has two such classes, the first Pontryagin class and the Euler class and the anomaly in this case is proportional to the Euler class. For an $\mathrm{SO}(4)$ field strength two-form F^{ab}, $a, b, = 1, 2, 3, 4$ the Euler class is

$$\chi(F) = \frac{1}{32\pi^2}\epsilon^{abcd} F^{ab} \wedge F^{cd} \tag{2.6}$$

which in terms of SU(2) field strengths is,

$$\frac{1}{4\pi^2}(trF_+^2 - trF_-^2)\,. \tag{2.7}$$

The descent procedure involves writing

$$\chi(F) = d\chi_3^{(0)}(A) \tag{2.8}$$

and the gauge variation as

$$\delta\chi_3^{(0)} = d\chi_2^{(1)}(A)\,. \tag{2.9}$$

The normal bundle anomaly for each SO(4) is then proportional to

$$\int_{\Sigma_2} \pi\chi^{(1)}(A) \tag{2.10}$$

with Σ_2 the self-dual string world-volume. (The factor of π appears instead of the usual 2π since the Fermions are Majorana and Weyl.) The total anomaly is

$$\int_{\Sigma_2} \pi Q_2(\chi_2^{(1)}(A_N) \mp \chi_2^{(1)}(A_T)) \tag{2.11}$$

with the sign correlated with the sign in (2.1). From now on we work with the upper sign in (2.1) and (2.11).

In the next section we will consider the self-dual strings in a (0,2) multiplet arising from the low energy description of fivebranes in the Coulomb branch. We will show how the anomaly may cancelled by an inflow mechanism and in doing so we will also see how the anomaly must scale with the number of fivebranes.

3. Anomaly cancellation

3.1 Generalities

There are various approaches and levels of analysis one can take in dealing with anomaly cancellation in string theory and M-theory, particularly in analysing anomalies for extended objects.

As usual in trying to cancel anomalies, one is free to add local counterterms to the lagrangian. In theories with UV divergences these can be viewed as part of the definition of the theory. In theories such as string theory, and presumably M-theory, such counterterms should in principle be computable in the underlying microscopic theory. Thus, in the original analysis of Green and Schwarz [21], anomaly cancellation was understood both by a direct string calculation, and in the low-energy effective lagrangian, and the local counterterms needed to cancel the anomaly in the low-energy effective theory could be verified directly. Similarly, the analysis of anomaly cancellation on D-branes in string theory requires certain anomalous, or Wess-Zumino, terms in the low-energy effective action on the D-brane, and it is possible to check that the required terms are present by a direct calculation [22] .

JHEP11(2004)015

The situation for NS 5-branes and M5-branes in IIA string theory and M-theory is less satisfactory. It was shown in [12] that the anomalies cancel for NS 5-branes after addition of a local counterterm to the 5-brane low-energy effective action, but to our knowledge this counterterm has not been verified by a direct calculation in string theory. Similarly, anomalies cancel for the M5-brane after one gives a careful definition of the C field and its action in the presence of an M5-brane [23]. This definition involves adding local counterterms to the M-theory low-energy effective action. There is however no microscopic derivation of these terms. However,the counterterm required in [12] does follow from the analysis of [23], so at least they are not independent problems [24].

In dealing with anomalies on extended objects there are two different points of view one can take. The NS 5-brane and M5-brane can be viewed as smooth soliton solutions of string theory or M-theory. Viewed this way, the zero modes localised on the 5-brane arise from a collective coordinate expansion of the bulk fields [25]–[27]. However, it has proved difficult to analyse anomalies directly in this framework because this involves dealing with the bulk Rarita-Schwinger equation in the non-trivial fivebrane geometry. The point of view which is most commonly taken is to split the degrees of freedom into bulk degrees of freedom and degrees of freedom localised on the brane without taking into account the detailed form of the brane soliton solution or the relation between localised zero modes and bulk fields. This is the point of view adopted in [23].

In this paper we will take this last point of view. As we discussed earlier, the M2-brane ending on an M5-brane can be viewed as a soliton solution to the M5-brane equations of motion, but for analysing anomalies we will treat the M2-brane and M5-brane zero modes as independent of the bulk fields. We also allow ourselves the freedom to add local counterterms to the lagrangian as long as they respect the symmetries of the system. As we discussed earlier, this means they should respect the $Spin(1,1) \times \mathrm{SO}(4)_T \times \mathrm{SO}(4)_N$ symmetry of the M2-M5 configuration.

3.2 Anomaly inflow for $Q_5 = 1$

The cancellation of the $\mathrm{SO}(4)_T$ anomaly in (2.11) is the most straightforward to understand and was already discussed in [13]. We redo this analysis here using the formalism developed in [23, 24].[1]

We denote the M5-brane and M2-brane world-volumes by Σ_6 and Σ_3 respectively. The M2-brane boundary on the M5-brane is the worldvolume of the self-dual string with worldvolume $\Sigma_2 \equiv \partial\Sigma_3$. We introduce two bump forms $d\rho(r)$ and $d\rho'(r')$ where r is the radial direction transverse to Σ_2 in Σ_6 and r' is the radial direction transverse to Σ_6 in the 11-dimensional spacetime manifold M_{11}. M-theory in M_{11} has, in the absence of M5-branes, a four-form field strength G_4 with $G_4 = dC_3$ and a Bianchi identity $dG_4 = 0$. As we mentioned earlier, the M5-brane worldvolume theory has, in the absence of self-dual strings, a three-form field strength h_3 with a Bianchi identity $dh_3 = -G_4|_{\Sigma_6}$. In the presence of M5-branes and self-dual strings the resulting Bianchi identities be-

[1] In the conventions of [13] we have chosen units with $T_3 = 1$, $\kappa_{11} = 1/T_6 = \pi$.

JHEP11(2004)015

come

$$\begin{aligned} dG_4 &= 2\pi\delta_5(\Sigma_6 \hookrightarrow M_{11}) \\ dh_3 &= \pi Q_2 \delta_4(\Sigma_2 \hookrightarrow \Sigma_6) - G_4|_{\Sigma_6} \,. \end{aligned} \tag{3.1}$$

Note the factor of π as opposed to 2π on the right hand side for the Bianchi identity of h_3. This arises because the flux quantisation for Dyonic strings in six dimensions is given by $eg = \pi n$ [28] . Physically, the quantities δ_5 and δ_4 in (3.1) are often thought of as delta functions with integral one in the spaces transverse to Σ_6 and Σ_2 respectively. However, a more careful mathematical treatment has turned out to be necessary in dealing with anomalies in the M5 system in which we think if these as the Poincare duals to Σ_6 in M_{11} and Σ_2 in Σ_6 respectively. Using the isomorphism between the Poincare dual and the Thom class of the normal bundle we can choose explicit representatives for δ_5 and δ_4 given by

$$\begin{aligned} \delta_5(\Sigma_6 \hookrightarrow M_{11}) &= d\rho'(r') \wedge \frac{e_4}{2} \\ \delta_4(\Sigma_2 \hookrightarrow \Sigma_6) &= d\rho(r) \wedge \frac{e_3}{2} \,, \end{aligned} \tag{3.2}$$

where e_4 is the global angular form with integral two over the S^4 fibres transverse to Σ_6 in M_{11}, and e_3 is the global angular form with integral two over the S^3 fibres transverse to Σ_2 in Σ_6.

Following [23, 24] we write

$$\frac{e_3}{2} = d\psi_2 + \Omega_3 \,, \tag{3.3}$$

with $d\Omega_3 = -\chi(F_T)$ and solve these Bianchi identities so that G_4 and h_3 are non-singular on Σ_6 and Σ_2 respectively:

$$\begin{aligned} G_4 &= dC_3 - 2\pi d\rho' \wedge \frac{e_3^{(0)}}{2} \\ h_3 &= db_2 - C_3 + \pi Q_2(\rho\Omega_3 - d\rho \wedge \psi_2) \,. \end{aligned} \tag{3.4}$$

Since h_3 must be gauge invariant, and the variation of Ω_3 under $\mathrm{SO}(4)_T$ gauge variations is $\delta\Omega_3 = d\chi_2^{(1)}(A_T)$, we learn using $\rho(0) = -1$ that b_2 has a $\mathrm{SO}(4)_T$ gauge variation so that the minimal coupling of the string to the two-form field on the M5-brane has a variation

$$\int_{\Sigma_2} \delta b_2 = \pi Q_2 \int_{\Sigma_2} \chi_2^{(1)} \tag{3.5}$$

which cancels the $\mathrm{SO}(4)_T$ anomaly in (2.11).

Where we differ from [13] is in the treatment of the $\mathrm{SO}(4)_N$ anomaly. In the above formalism $G_4|_{\Sigma_6} = dC_3|_{\Sigma_6}$ and there does not seem to be room for the additional $\mathrm{SO}(4)_N$ variation of b_2 found in [13].

Fortunately, the $\mathrm{SO}(4)_N$ anomaly can be cancelled by adding a local counterterm to the M5-brane action:

$$S_N = -\int_{\Sigma_6} h_3 \wedge \chi^{(0)}(A_N) \,. \tag{3.6}$$

It is easy to see using the second equation in (3.1) that this term has an anomalous variation localised on Σ_2 that cancels the anomaly in (2.11). In the following section we will see that a coupling of the form (3.6) is already know to exist in the theory of $Q_5 > 1$ M5-branes.

We have thus demonstrated anomaly cancellation for Q_2 M2-branes ending on a single M5-brane. For $Q_5 > 1$ M5-branes we do not know how to carry out such a general analysis, because neither the fermion zero mode structure nor the Q_5 dependence of the coupling (3.6) is known. However, we can obtain some partial results by utilising a known generalisation of the coupling (3.6) which appears on the Coulomb branch of the theory with $Q_5 > 1$.

3.3 Anomaly inflow for $Q_5 > 1$

The analysis in the previous section involved coupling to the U(1) centre of mass multiplet of the $(2,0)$ theory. We now consider a general M5-brane theory with an ADE symmetry $G \times \mathrm{U}(1)$ broken down to $(H \times \mathrm{U}(1)) \times \mathrm{U}(1)$.

As in the analogous D1-D3 system, it is expected that this theory will have self-dual strings which couple to the $\mathrm{U}(1) \subset G$ factor. These are the analogs of the non-abelian 't Hooft-Polyakov monopoles in the D1-D3 system. Assuming that such solutions exist and act as sources for the h field in the relative U(1) multiplet, we can deduce some facts about the anomalies of the fermion zero modes on such strings.

The coupling in the five-brane theory which is anomalous in such a string background originates from a coupling derived in [15, 16]. Following the conventions in [16] the coupling is given by

$$S_N = \alpha_e \int_{\Sigma_6} h_3 \wedge \Omega_3(\hat{\phi}, A)\,, \tag{3.7}$$

where

$$\begin{aligned} d\Omega_3 = \frac{1}{64\pi^2}\epsilon_{a_1\ldots a_5}[(D\hat{\phi})^{a_1} \wedge (D\hat{\phi})^{a_2} \wedge (D\hat{\phi})^{a_3} \wedge (D\hat{\phi})^{a_4} - \\ - 2F^{a_1a_2} \wedge (D\hat{\phi})^{a_3} \wedge (D\hat{\phi})^{a_4} + F^{a_1a_2} \wedge F^{a_3a_4}]\hat{\phi}^{a_5} \end{aligned} \tag{3.8}$$

with $(D_i\phi)^a = \partial_i\phi^a - A_i^{ab}\phi^b$ the covariant derivative of ϕ^a with A_i^{ab} the SO(5) gauge field of the five-brane normal bundle and $\hat{\phi}^a = \frac{\phi^a}{\|\phi\|}$. The coupling constant α_e depends on the breaking of the five-brane theory gauge group. If G is the ADE Lie algebra labelling the $(0,2)$ theory, then for a breaking given by $G \to H \times \mathrm{U}(1)$, it was argued that

$$\alpha_e = \frac{1}{4}(|G| - |H| - 1)\,. \tag{3.9}$$

For example if $G = \mathrm{SU}(Q_5+1)$ and $H = \mathrm{SU}(Q_5)$ then, $\alpha_e = \frac{1}{2}Q_5$.

This coupling was derived by considering anomalies in the Coulomb branch of the five-brane theory, ie. when the scalars have non-zero vacuum expectation values. The point is that the U(1) multiplet at low energies must include an interaction term to compensate for what would otherwise be a difference in the anomaly at the origin of moduli space and the anomaly at a generic point in the Coulomb branch. Hence the decoupling of the

U(1) multiplet never really happens; this term is always sensitive to the full theory and so even in the infrared there remains some information of the integrated out ultra-massive modes.

Here we will evaluate this interaction term in the presence of a self-dual string embedded in the naively decoupled U(1) tensor multiplet.

For the self-dual string solution we can take just one scalar, say ϕ^5, to be non-zero and obeying (2.1), while also taking $A^{5a}=0$ to reduce the SO(5) to the SO(4) preserved by the string solution. Then it is easy to see that in the presence of the self-dual string Ω_3 reduces to $\chi^{(0)}(A)/2$. The effective coupling on the fivebrane in the presence of the self-dual string is thus

$$S_N = -\frac{\alpha_e}{2}\int_{\Sigma_6} h_3\wedge\chi^{(0)}(A_N)\,. \tag{3.10}$$

In the presence of the self-dual string the coupling (3.10) is not gauge invariant. Rather, its gauge variation is

$$\delta S_N = -\frac{\alpha_e}{2}\int_{\Sigma_6} h_3\wedge d\chi^{(1)} = -\frac{\alpha_e}{2}\int_{\Sigma_6} dh_3\wedge\chi^{(1)}\,. \tag{3.11}$$

Now the self-dual string acts as a source of h_3 via the equation

$$dh_3 = \pi Q_2\delta_4(\Sigma_2\hookrightarrow\Sigma_6) \tag{3.12}$$

Using this, the variation (3.11) becomes

$$-\frac{1}{2}\pi\alpha_e Q_2\int_{\Sigma_2}\chi^{(1)}\,. \tag{3.13}$$

Now, assuming the anomalous variation is, as before, cancelled by the fermion zero modes we deduce that the $\mathrm{SO}(4)_N$ zero mode anomaly of the self-dual string in the Coulomb branch scales as:

$$c = \frac{1}{2}\alpha_e Q_2\,. \tag{3.14}$$

Using the above and the equation for α_e, (3.9) we may compute the anomaly for the string, that is c in terms of the charges Q_2, Q_5.

Consider the case of pulling off a single five brane from a stack of Q_5+1 fivebranes and embedding the string in the relative U(1). The five-brane theory would have $G=\mathrm{SU}(Q_5+1)$ and $H=\mathrm{SU}(Q_5)$ giving, $\alpha_e=\frac{1}{2}Q_5$. This would then give

$$c = \frac{1}{4}Q_2Q_5\,. \tag{3.15}$$

We may then interpret c as being related to the number of degrees of freedom of the string. The Q_2Q_5 dependence of the anomaly is then consistent with the cross section scattering calculation of the self-dual string described in [7] .

A more interesting situation arises if one considers a different breaking pattern for the five-branes. Take a stack and separate all the branes thus giving a maximal breaking with $G=\mathrm{SU}(Q_5+1)$ and $H=\mathrm{U}(1)^{Q_5}$. In this case simply applying the above formula yields,

$c = \frac{1}{8}Q_2(Q_5^2 + Q_5 - 1)$. In this case no cross section scattering calculation has been done to confirm the charge dependence. It would be interesting to find other ways to study this system which would confirm this behaviour.

Note that for this calculation we are really only using the supersymmetry/R-symmetry preserved by the string solution to determine the anomaly in conjunction with the Ganor, Motl, Intrilligator term (3.8) for the cancellation and so the precise form of the solution should not matter. Thus even if one might be concerned about applying the solution of [6] in the more exotic circumstances described above, provided the symmetries of the string are the same our results should remain valid.

4. Dimensional reduction

We now consider the implications of these terms for IIA string theory by reducing M theory on an S^1. To be explicit we take the M5-worldvolume to lie in the $(0, 1, \cdots 5)$ plane and the M2-worldvolume to lie in the (016) plane. The self-dual string worldvolume then lies in the (01) plane. Normal bundle gauge transformations that preserve this configuration act on the $(2, 3, 4, 5)$ coordinates ($\mathrm{SO}(4)_T$) or the $(7, 8, 9, 10)$ coordinates ($\mathrm{SO}(4)_N$).

There are then two interesting reductions to IIA string theory. We can take one of the $(2, 3, 4, 5)$ coordinates to be periodic. This turns the M2 into a D2-brane in IIA theory and the M5 which wraps this periodic coordinate into a D4-brane. The $\mathrm{SO}(4)_T$ symmetry is broken to SO(3) which has no Euler class, so there is no SO(3) normal bundle anomaly. However, the $\mathrm{SO}(4)_N$ symmetry is preserved and has an anomaly derived by descent from the Euler class.

Cancellation of this normal bundle anomaly for a D2 ending on a D4 requires a coupling on the D4-worldvolume of the form

$$\int_{\Sigma^5} F_2 \wedge \chi^{(0)}(A_N)\,, \tag{4.1}$$

where F_2 is the U(1) gauge field strength on the D4-brane. Note that this term is distinct from the usual anomalous couplings on D-branes [9]–[11]. Because it is independent of the bulk Ramond-Ramond fields, it would arise at one-loop level rather than as a tree-level coupling, presumably explaining why it has not been seen in previous explicit calculations of anomalous couplings [22]and [29]–[31]. In fact, closely related couplings (but for relative U(1) factors) have been discussed in [18] and [19].

The other inequivalent reduction takes one of the $(7, 8, 9, 10)$ coordinates to be periodic. This turns the M2 into a D2 while converting the M5 into a NS5-brane. Now the $\mathrm{SO}(4)_N$ symmetry is broken to SO(3) with vanishing anomaly while the $\mathrm{SO}(4)_T$ symmetry is preserved. Cancellation of this normal bundle anomaly then requires a coupling on the NS5-brane worldvolume analogous to (3.10) .

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JHEP11(2004)015

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